Exploration of Linear, Quadratic, and Cubic Functions

1. Introduction

Functions describe relationships between variables. This exploration will examine linear, quadratic, and cubic functions - key tools in algebra, physics, economics, and data modeling.

2. Linear Functions

A linear function has the form:

$$f(x) = mx + b$$

Where:

- m is the slope (rate of change)
- b is the y-intercept

Key Characteristics:

- Graph: straight line
- Constant rate of change
- Domain and range: all real numbers

Example:

$f(x) = 2x + 3$

Slope = 2, y-intercept = 3

Real-World Application:

Earnings over time: E(x) = 20x represents $20 earned per hour.

3. Quadratic Functions

A quadratic function has the form:

$$f(x) = ax^2 + bx + c \quad \text{where } a \neq 0$$

Key Characteristics:

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- Graph: parabola (U-shaped)
- Vertex: maximum or minimum point
- Axis of symmetry: $x = -b / (2a)$
- Can have 0, 1, or 2 real roots

Example:

$f(x) = x^2 - 4x + 3$

Vertex: (2, -1), Roots: $x = 1$ and $x = 3$

Real-World Application:

Projectile motion: $h(t) = -5t^2 + 20t + 1$ models height over time.

4. Cubic Functions

A cubic function has the form:

$f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$

Key Characteristics:

- Graph: S-curve or N-curve
- May have 1 or 3 real roots
- May have an inflection point
- End behavior depends on sign of a

Example:

$f(x) = x^3 - 6x^2 + 11x - 6$

Roots: $x = 1, 2, 3$ (factored as $(x-1)(x-2)(x-3)$)

Real-World Application:

Used in economics for revenue-cost analysis or in physics for modeling torque.

5. Comparison Table

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Feature	Linear	Quadratic	Cubic
Degree	1	2	3
Graph Shape	Line	Parabola	S or N curve
Max Real Roots	1	2	3
Turning Points	0	1 (vertex)	Up to 2
Rate of Change	Constant	Linear variation	Quadratic variation

6. Conclusion

Each function type - linear, quadratic, and cubic - provides powerful ways to represent and predict behavior. They build the foundation of algebra and help us analyze data, design systems, and understand the world.

Sources

Sources:

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