Golden Ratio Exploration

the origin of perfection

By: Felipe Garcia

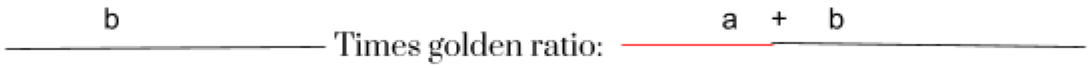
Introduction

The human race has been attracted to the golden ratio as far back as ancient Greece. The golden ratio, a constante also know the greek letter ϕ (phi), has fascinated scholars, artists, architects, and scientists. Defined as $\frac{1+\sqrt{5}}{2}$ it appears in multiple fields from art, nature, and architecture to geometry, and algebra. The golden ratio is frequently observed in the proportions of natural objects, classical art (Da Vinci), the human body, and even used by famous architects such as Le Corbusier to obtain harmonious designs leading to the belief that it represents an idea of balance, symmetry, and aesthetics.

In this investigation the golden ratio shall be explored across all fields, including its derivation through algebra and geometry, its aesthetically pleasing appearance through design and art, and finally its broader meaning. Specifically I will prove its value through the quadratic formula, analyze its correlation with the Fibonacci sequences, and examine real-world applications that highlight its importance. Through this exploration, I aim to understand why this unique proportion has caught the eye of mathematicians, and artists for millennia and whether its influence is as universal as often stated.

Definition of The Golden Ratio

The golden ratio is defined by the positive number ϕ, such that when a line is divided into two segments, the ratio of the whole line to the longer segment is the same as the ratio of the longer segment to the shorter segment.



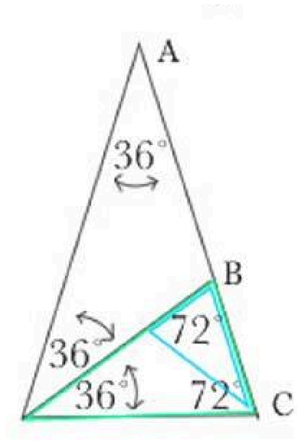
$$\frac{a+b}{b} = \frac{a}{b} = \phi \quad \Rightarrow \quad \phi = 1 + \frac{1}{\phi} \quad \Rightarrow \quad \phi^2 = \phi + 1 \quad \Rightarrow \quad \phi^2 - \phi - 1 = 0$$

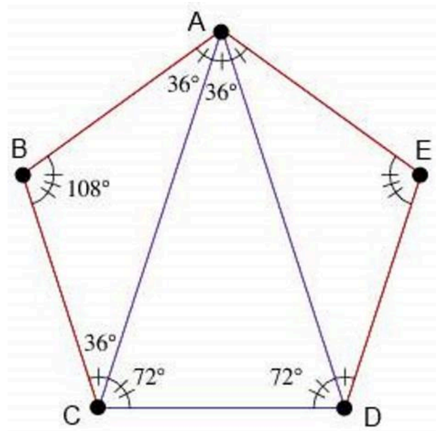
Apply quadratic formula: $\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2} \quad \Rightarrow \quad \frac{1\pm\sqrt{1+4}}{2} \quad \Rightarrow \quad \frac{1\pm\sqrt{5}}{2}$ since ϕ is a positive number rule out the possibility of - therefore $\phi = \frac{1+\sqrt{5}}{2}$

ϕ In Geθmɛtry

The golden ratio is vastly useful when applied to shapes, unabling the possibility to create other shapes, exemplified by the golden triangle towards pentagons and pentagrams. Golden shapes have interesting useful patterns and can be seen around the world.

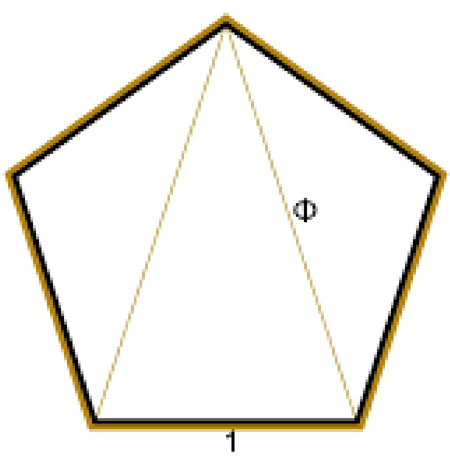
The golden triangle is an isosceles triangle where the ratio of the longer side to the shorter side is ϕThus its angles are 36, 72 and 72 commun in the pentagon and pentagram constructions. A regular pentagon contains multiple instances of the golden ratio, the ratio of its side to its diagonal is phi.

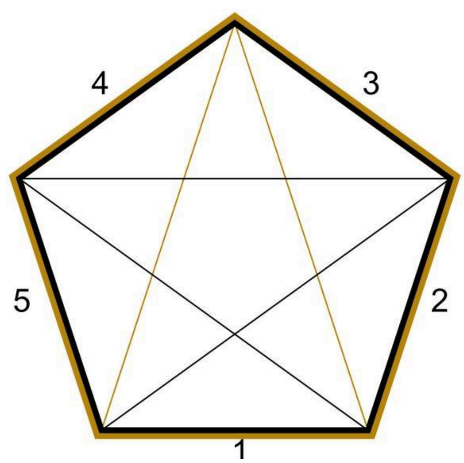




$CA = \phi \quad BC = 1$

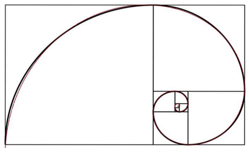
Now the pentagram is built by joining the vertices of a pentagon only through diagonal lines, therefore made up entirely by golden triangles.

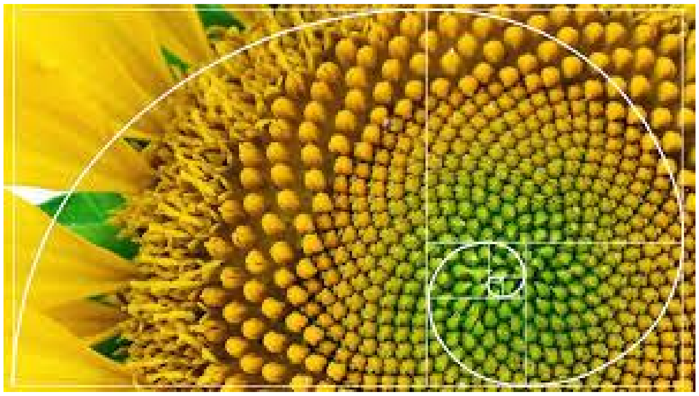


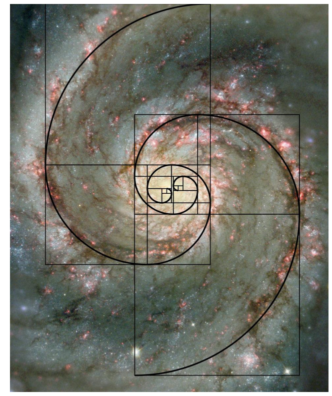


$Sides \ = \ 1 \qquad Diagonals \ = \ \phi$

As pentagonal symmetry appears in flowers, fruits, starfish, etc the golden number is often linked to nature generating a sense of curiosity.

Golden shapes have other features, such as the golden rectangle. The sides of a golden rectangle have ratio of $\phi: 1$. What is truly amusing is that if you were to remove a square from the golden rectangle, the result would be another golden rectangle.  This process creates the Fibonacci spiral seen in everything, from seeds in a plant to something as astronomical as the universe.





The fibonacci sequence. One of the most famous properties of the fibonacci sequence is that if you take two consecutive Fibonacci numbers and divide them, the result gets closer and closer to the golden ratio.

$Fibonacci\ sequence$: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

$lim\ n \to \infty \frac{Fn}{Fn-1} = \phi \ = \frac{1+\sqrt{5}}{2}$

To see why this happens, consider the Fibonacci recurrence relation, the base for the sequence itself, the recurrence relation states that two consecutive fibonacci numbers added will add to the next number:

$Fn = Fn - 1 + Fn - 2.$

If we assume that the ratio $r = \frac{Fn}{Fn-1}$ becomes constant for large n, we can rewrite the recurrence as:

$r = 1 + \frac{1}{r}$ $\Rightarrow$ $r^2 = r + 1$ $\Rightarrow$ $r = \frac{1\pm\sqrt{5}}{2}$

Since we are dealing with ratios of positive numbers, we take the positive root:

$r = \frac{1+\sqrt{5}}{2} = \phi$, therefore $\frac{Fn}{Fn-1}$ converges to ϕ.

What’s truly fascinating is that although Fibonacci introduced this sequence in 1202, the connection to the golden ratio wasn’t uncovered until over 500 years later—in the 18th century—by mathematicians Daniel Bernoulli and Abraham de Moivre. They were among the first to express the nnnth Fibonacci number using an explicit formula involving the golden ratio, revealing a deep and elegant link between algebra and number theory.

All Around Us

It's no surprise that the golden ratio is often associated with perfection its influence is evident in countless everyday objects and designs around us. Many international companies and designers use the golden ratio to enhance the visual appeal and harmony of their products and branding. This mathematical proportion isn't just a theoretical concept; it's a practical tool for creating beauty and balance in the modern world.

One of the first places I personally noticed the golden ratio was on my own debit card. Curious, I decided to test my hypothesis, measuring the card’s sides and sure enough, the ratio held. That discovery sparked a wave of exploration. I began measuring playing cards, notebooks, book covers, and other household items. To my amazement, the golden ratio appeared again and again, subtly guiding the dimensions of things we often take for granted.

Major companies have also embraced this principle in their branding and product design. Apple, for instance, uses the golden ratio both in the structure of its logo and the layout of devices like the iPhone and iPad. The logo’s curves align with circles derived from the golden ratio, and the clean, intuitive proportions of their products enhance the user experience while maintaining an elegant aesthetic.

Luxury fashion brands such as Gucci, Chanel, and Louis Vuitton are also known for applying the golden ratio in their logo design, packaging, and even clothing collections, aiming to evoke a sense of refinement and timeless beauty. The golden ratio has also influenced the design of credit cards, business cards, furniture, and architecture from the Parthenon in ancient Greece to modern skyscrapers.

In photography and visual arts, the golden ratio is often used as a compositional guideline through the Phi Grid or the Golden Spiral to achieve balance and draw the viewer's eye naturally through the frame. Even in web design and user interfaces, designers use golden proportions to create intuitive and visually appealing layouts.

From art museums to store shelves, and from smartphones to spiral galaxies, the golden ratio quietly shapes the world around us a hidden formula for beauty that links mathematics with the way we see and feel.

Conclusion

Through this exploration, I've come to understand why the golden ratio has captivated mathematicians, artists, and thinkers for centuries — and now, it has captured my own attention as well. I'm genuinely amused and fascinated by how a single number can appear so naturally in places ranging from ancient architecture and classical art to modern design and everyday objects. Its presence in geometry, algebra, nature, and even branding shows that it represents far more than just a mathematical constant — it reflects an underlying harmony that connects logic with beauty. Investigating the golden ratio has not only deepened my appreciation for mathematics, but also reminded me of how powerful and inspiring math can be when we look at it beyond the classroom.